BOSE SYMMETRY INTERFERENCE EFFECTS OF 4π FINAL STATES

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Abstract

We carefully analyze the relative branching ratios of 4π final states $\pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$ and $\pi^0\pi^0\pi^0\pi^0$, from various resonances of $J^{PC} = 0^{++}$, 0^{-+}, 2^{++}. We find that the Bose symmetry interference effects would make their ratios to obviously differ from the naive counting values without considering these effects. The results should be applied to estimate correctly various 4π decay branching ratios of relevant resonances.

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I. Introduction

In the energy range of $1 \sim 2.5$ GeV, there exist very rich hadronic resonance spectra, including possible 0^{++}, 0^{-+}, and 2^{++} glueballs. An important source of information about the nature of these resonances is their various decay branching ratios. Among the observed decay modes for the 0^{++}, 0^{-+} and 2^{++} resonances, the 4π final state is a very important one[1].

There are three kinds of 4π final states: $\pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$ and $\pi^0\pi^0\pi^0\pi^0$. Usually, due to specialty and limitation of each detector and other reasons, one experiment is only good at studying one kind of the 4π final states. For example, the Crystal Barrel (CBAR) detector is particularly good at studying neutral final states and therefore studied resonances

decaying into $\pi^0\pi^0\pi^0\pi^0$ final state[2]; the BES (Beijing Spectrometer) detector is good at detecting charged particles and only has studied J/Ψ radiative decaying into $\pi^+\pi^-\pi^+\pi^-$ final state[3]. The problem is that from the measured rate for one kind of the 4π final states how to deduce the rates for other two kinds of the 4π final states.

The 4π final states are usually produced via 2-meson intermediate states M_1 and M_2, namely the parent-resonance decays into two mesons which then result in the 4π final states:

$$M \to M_1 + M_2 \to 4\pi.$$

The parent-meson which we are interested in are $0^{++}, 0^{-+}$ and 2^{++} etc., because they have the same quantum numbers as glueballs which are under intensive discussions at present. M_1 and M_2 can be various mesons among which $\sigma\sigma, \rho\rho$ and $f_2\sigma$ are the most possible candidates[2, 3] and therefore discussed in this work.

Naive counting for the 4π final states from simple isospin decomposition would result in

$$\begin{aligned}
&\Gamma(M \to \pi^+\pi^-\pi^0\pi^0) : \Gamma(M \to \pi^+\pi^-\pi^+\pi^-) : \Gamma(M \to \pi^0\pi^0\pi^0\pi^0) \\
&= 4 : 4 : 1, \quad \text{for } \mathrm{f}_2\sigma, \sigma\sigma \text{ intermediate states;} \qquad (1) \\
&\Gamma(M \to \pi^+\pi^-\pi^0\pi^0) : \Gamma(M \to \pi^+\pi^-\pi^+\pi^-) : \Gamma(M \to \pi^0\pi^0\pi^0\pi^0) \\
&= 2 : 1 : 0, \quad \text{for } \rho\rho \text{ intermediate states,} \qquad (2)
\end{aligned}$$

where all interference effects are neglected.

Since all pions can be treated as identical particles and each mode has the same production amplitude up to an SU(2) factor, so unless all the interference terms cancel each other after integration over the invariant phase space of final states, their effects can be important. In earlier literatures, the naive counting was employed to evaluate production rate of one mode from others. Even though this counting way is simple and valid in certain cases, it may bring up remarkable errors in some cases.

In this work, we carefully analyze the relative ratios of $B(M \to \pi^+\pi^-\pi^0\pi^0) : B(M \to \pi^+\pi^-\pi^+\pi^-) : B(M \to \pi^0\pi^0\pi^0\pi^0)$ by including interference terms precisely for various masses of the parent meson of $0^{++}, 0^{-+}, 2^{++}$, and intermediate 2-meson states.

In Sec.II, we present the formulation for analysis; the numerical results and discussion are given in Sec.III.

II. Formulation

For $M \to \pi_1(p_1)\pi_2(p_2)\pi_3(p_3)\pi_4(p_4)$ where $p_i's$ $(i = 1...4)$ are the four-momenta of the four produced pions and we use notation

$$p_{ab} \equiv p_a + p_b, \quad a, b = 1, ..., 4, \text{ and } a \neq b.$$

The propagators take the Breit-Wigner form [4, 5]

$$\begin{aligned}
F_{ab} &= \frac{-i}{p_{ab}^2 - m^2 + i\Gamma m} \quad \text{for scalar mesons;} \\
D_{ab}^{\alpha\beta} &= \frac{i}{p_{ab}^2 - m^2 + i\Gamma m}\tilde{g}^{\alpha\beta} \quad \text{for massive vector;} \\
D_{ab}^{\alpha\beta\gamma\delta} &= \frac{-i}{p_{ab}^2 - m^2 + i\Gamma m}[\frac{1}{2}(\tilde{g}^{\alpha\gamma}\tilde{g}^{\beta\delta} + \tilde{g}^{\alpha\delta}\tilde{g}^{\beta\gamma}) - \frac{1}{3}\tilde{g}^{\alpha\beta}\tilde{g}^{\gamma\delta}] \\
&\quad \text{for spin} - 2 \text{ tensor meson,} \qquad (3)
\end{aligned}$$

where

$$\tilde{g}^{\alpha\beta} \equiv -g^{\alpha\beta} + \frac{p_{ab}^\alpha p_{ab}^\beta}{m^2}, \qquad (4)$$

and m is the mass of the concerned intermediate meson of spin 0 or 1 or 2. In the following we explicitly present the expressions with $M \to M_1 + M_2 \to 4\pi$ for various M, M_1, M_2 identities.

1. Decay of $M(0^{++}) \to 4\pi$.

To investigate the interference effects of $M \to 4\pi$, we distinguish the processes caused by different intermediate states. Here we first ignore possible interferences between $M \to \sigma\sigma \to 4\pi$ and $M \to \rho\rho \to 4\pi$ and later we will show that except for special cases, it is legitimate. Then we argue that the conclusion can be generalized to most situations.

(a) The squares of amplitudes corresponding to $\sigma\sigma$ intermediate state are

$$|M|^2 = \frac{g^2}{2}|F_{12}^\sigma F_{34}^\sigma|^2 \quad \text{for } f_0 \to \pi^+\pi^-\pi^0\pi^0;$$

$$|M|^2 = \frac{g^2}{4}|F^\sigma_{12}F^\sigma_{34}+F^\sigma_{14}F^\sigma_{32}|^2 \quad \text{for } f_0 \to \pi^+\pi^-\pi^+\pi^-;$$
$$|M|^2 = \frac{g^2}{24}|F^\sigma_{12}F^\sigma_{34}+F^\sigma_{13}F^\sigma_{24}+F^\sigma_{14}F^\sigma_{32}|^2 \quad \text{for } f_0 \to \pi^0\pi^0\pi^0\pi^0. \tag{5}$$

(b) via $\rho\rho$ intermediate states.

$$|M|^2 = \frac{g'^2}{2}|(p_1-p_2)\cdot(p_3-p_4)F^\rho_{12}F^\rho_{34}+(p_1-p_4)\cdot(p_3-p_2)F^\rho_{14}F^\rho_{32}|^2$$
$$\text{for } f_0 \to \pi^+\pi^0\pi^-\pi^0;$$
$$|M|^2 = \frac{g'^2}{4}|(p_1-p_2)\cdot(p_3-p_4)F^\rho_{12}F^\rho_{34}+(p_1-p_4)\cdot(p_3-p_2)F^\rho_{14}F^\rho_{32}|^2$$
$$\text{for } f_0 \to \pi^+\pi^-\pi^+\pi^-, \tag{6}$$

where $p_i's$ are the momenta of the outgoing pions and $F^\sigma_{ij}, F^\rho_{ij}$ are the propagators of $\sigma-$meson and $\rho-$meson respectively. It is noted that there is no process $f_0 \to \rho\rho \to 4\pi^0$, because $\rho^0 \to \pi^0\pi^0$ is forbidden by isospin symmetry.

2. Decay of 0^{-+} mesons.

It is obvious that $0^{-+} \to \sigma\sigma$ is forbidden by parity and angular-momentum conservations, it decays into 4π only via $\rho\rho$ intermediate states.

$$|M|^2 = \frac{g'^2}{2}|\epsilon_{\mu\nu\lambda\rho}(p_1^\mu p_2^\nu p_3^\lambda p_4^\rho F^\rho_{12}F^\rho_{34} - p_1^\mu p_4^\nu p_3^\lambda p_2^\rho F^\rho_{14}F^\rho_{32})|^2$$
$$\text{for } 0^{-+} \to \pi^+\pi^0\pi^-\pi^0;$$
$$|M|^2 = \frac{g'^2}{4}|\epsilon_{\mu\nu\lambda\rho}(p_1^\mu p_2^\nu p_3^\lambda p_4^\rho F^\rho_{12}F^\rho_{34} - p_1^\mu p_4^\nu p_3^\lambda p_2^\rho F^\rho_{14}F^\rho_{32})|^2$$
$$\text{for } 0^{-+} \to \pi^+\pi^-\pi^+\pi^-. \tag{7}$$

3. Decay of 2^{++} mesons.

(a) Via $\sigma\sigma$ intermediate states,

$$|M|^2 = \frac{g''^2}{2}|\sqrt{\frac{1}{6}}(-r^2+3r_Z^2)F^\sigma_{12}F^\sigma_{34}|^2$$
$$\text{for } 2^{++} \to \pi^+\pi^-\pi^0\pi^0;$$
$$|M|^2 = \frac{g''^2}{4}|\sqrt{\frac{1}{6}}(-r^2+3r_Z^2)F^\sigma_{12}F^\sigma_{34}+\sqrt{\frac{1}{6}}(-r''^2+3r''^2_Z)F^\sigma_{14}F^\sigma_{32}|^2$$
$$\text{for } 2^{++} \to \pi^+\pi^-\pi^+\pi^-;$$

$$
\begin{aligned}
|M|^2 &= \frac{1}{24}g^{''2}|\sqrt{\frac{1}{6}}(-r^2+3r_Z^2)F^\sigma_{12}F^\sigma_{34}+\sqrt{\frac{1}{6}}(-r^{'2}+3r_Z^{'2})F^\sigma_{13}F^\sigma_{24}+ \\
&\quad \sqrt{\frac{1}{6}}(-r^{''2}+3r_Z^{''2})F^\sigma_{14}F^\sigma_{32}|^2 \quad \text{for } 2^{++}\to\pi^0\pi^0\pi^0\pi^0,
\end{aligned} \tag{8}
$$

where

$$
r\equiv p_{12}-p_{34},\quad r^{'}\equiv p_{13}-p_{24},\quad r^{''}\equiv p_{14}-p_{32}.
$$

(b) Via $\rho\rho$ intermediate states.

For 2^{++} decays, $\sigma\sigma$ production can only occur at d-wave, which is more suppressed, therefore, the $\rho\rho$ mode may dominate in the $2^{++}\to 4\pi$ decays. The amplitudes are

$$
\begin{aligned}
|M|^2 &= \frac{g^{''2}}{2}|(-p^x_{12}p^x_{34}-p^y_{12}p^y_{34}+2p^z_{12}p^z_{34})F^\rho_{12}F^\rho_{34}+(-p^x_{14}p^x_{32}-p^y_{14}p^y_{32}+2p^z_{14}p^z_{32})F^\rho_{14}F^\rho_{32}|^2 \\
&\quad \text{for } 2^{++}\to\pi^+\pi^0\pi^-\pi^0; \\
|M|^2 &= \frac{g^{''2}}{4}|(-p^x_{12}p^x_{34}-p^y_{12}p^y_{34}+2p^z_{12}p^z_{34})F^\rho_{12}F^\rho_{34}+(-p^x_{14}p^x_{32}-p^y_{14}p^y_{32}+2p^z_{14}p^z_{32})F^\rho_{14}F^\rho_{32}|^2 \\
&\quad \text{for } 2^{++}\to\pi^+\pi^-\pi^+\pi^-.
\end{aligned} \tag{9}
$$

4. Decay of $2^{++}\to 4\pi$ via $f_2\sigma$ intermediate states.

Here we have a simplified expression for the decay modes instead of using the propagator given in eqs.(3) as

$$
|M|^2 = |(-T^{11}-T^{22}+2T^{33})_{(ab)}F_{f_2}(ab)F_\sigma(cd)|^2, \tag{10}
$$

where

$$
T^{ii} = q^iq^i+\frac{1}{3}(1+p^ip^i/M^2_{f_2})|\vec{q}|^2, \tag{11}
$$

and

$$
p_{ab}=p_a+p_b,\quad q_{ab}=p_a-p_b,\quad F_{f_2}=\frac{1}{M^2_{f_2}-s_{ab}-iM_{f_2}\Gamma_{f_2}},\quad s_{ab}=p^2_{ab}.
$$

The subscript (ab) denotes the argument indices in the tensor T^{ii}, Thus

$$
\begin{aligned}
|M|^2 &= \frac{1}{2}|(-T^{11}-T^{22}+2T^{33})_{12}F_{f_2}(12)F_\sigma(34)|^2 \\
&\quad \text{for } 2^{++}\to f_2\sigma\to\pi^+\pi^-\pi^0\pi^0; \\
|M|^2 &= \frac{1}{4}|(-T^{11}-T^{22}+2T^{33})_{12}F_{f_2}(12)F_\sigma(34)+(-T^{11}-T^{22}+2T^{33})_{14}F_{f_2}(14)F_\sigma(32)|^2 \\
&\quad \text{for } 2^{++}\to f_2\sigma\to\pi^+\pi^-\pi^+\pi^-;
\end{aligned}
$$

$$|M|^2 = \frac{1}{24}|(-T^{11}-T^{22}+2T^{33})_{12}F_{f_2}(12)F_\sigma(34)+(-T^{11}-T^{22}+2T^{33})_{13}F_{f_2}(13)F_\sigma(24)+ (-T^{11}-T^{22}+2T^{33})_{14}F_{f_2}(14)F_\sigma(32)|^2$$

$$\text{for } 2^{++}\to f_2\sigma\to\pi^0\pi^0\pi^0\pi^0. \tag{12}$$

5. The interference between channels with $\sigma\sigma$ and $\rho\rho$ intermediate states in $M\to 4\pi$.

(a) Above, we have ignored possible interference between channels with $\sigma\sigma$ and $\rho\rho$ intermediate states for the 4π final states, just because we assume that one of the two modes would overwhelm over the other. This allegation might deviate from reality. So in this subsection, we study this interference effects. As an example, we only concentrate on the 0^{++} decays. Then we have the squares of amplitudes as

$$\begin{aligned}
|M|^2 &= \frac{1}{2}|g^{'}(p_1-p_2)\cdot(p_3-p_4)F^\rho_{12}F^\rho_{34}+g^{'}(p_1-p_4)\cdot(p_3-p_2)F^\rho_{14}F^\rho_{32}+gF^\sigma_{13}F^\sigma_{24}|^2\\
&\quad \text{for } f_0\to\pi^+\pi^0\pi^-\pi^0;\\
|M|^2 &= \frac{1}{4}|g^{'}(p_1-p_2)\cdot(p_3-p_4)F^\rho_{12}F^\rho_{34}+g^{'}(p_1-p_4)\cdot(p_3-p_2)F^\rho_{14}F^\rho_{32}+g(F^\sigma_{12}F^\sigma_{34}+F^\sigma_{14}F^\sigma_{32})|^2\\
&\quad \text{for } f_0\to\pi^+\pi^-\pi^+\pi^-;\\
|M|^2 &= \frac{1}{24}|gF^\sigma_{12}F^\sigma_{34}+gF^\sigma_{13}F^\sigma_{24}+gF^\sigma_{14}F^\sigma_{32}|^2\\
&\quad \text{for } f_0\to\pi^0\pi^0\pi^0\pi^0.
\end{aligned} \tag{13}$$

(b) As an illustration let us study the decay of $f_0(1750)$, because there are data available for $f_0(1750)\to\rho\rho$ and $\sigma\sigma$ [6]. The partial decay widths are

$$\begin{aligned}
\Gamma(f_0\to\sigma\sigma) &= \frac{g_f^2}{16\pi M_f}(1-\frac{4m_\sigma^2}{M_f^2})^{1/2},\\
\Gamma(f_0\to\rho\rho) &= \frac{g_f^{'2}}{16\pi M_f}(1-\frac{4m_\rho^2}{M_f^2})^{1/2}[3+\frac{1}{4m_\rho^4}(M_f^4-4M_f^2m_\rho^2)],\\
\Gamma(\rho\to 2\pi) &= \frac{g_\rho^2 m_\rho}{48\pi}(1-\frac{4m_\pi^2}{m_\rho^2})^{3/2},\\
\Gamma(\sigma\to\pi\pi) &= \frac{g_\sigma^2}{16\pi m_\sigma}(1-\frac{4m_\pi^2}{m_\sigma^2})^{1/2}.
\end{aligned} \tag{14}$$

So we have

$$g\equiv g_f\cdot g_\sigma^2,\qquad g^{'}\equiv g_f^{'}\cdot g_\rho^2. \tag{15}$$

By setting $\Gamma^{th} = \Gamma^{exp}$, we have obtained all the coupling constants straightforwardly.

III. Numerical results and discussion

We have employed the Monte-Carlo program to carry out the calculations of the widths. And in the practical calculation, we need to multiply the propagators of vector and tensor F_ρ and F_{f_2} in all the equations by the Blatt-Weisskopt barrier factor $B_l(p)$[5, 7] ($B_0(p) = 1$), which is widely used in partial-wave analyses. Namely, in our program the F_ρ is replaced by $F_\rho B_l(p)$ and F_{f_2} by $F_{f_2} B_2(p)$ respectively. For the $\sigma-$propagator, there are various forms. Here we only use two typical ones. The first is [8],

$$F_\sigma = \frac{1}{M_\sigma^2 - s - iM_\sigma(\Gamma_1(s) + \Gamma_2(s))}, \tag{16}$$

where

$$\Gamma_1(s) = G_1 \frac{\sqrt{1-4m_\pi^2/s}}{\sqrt{1-4m_\pi^2/M_\sigma^2}} \frac{s - m_\pi^2/2}{M_\sigma^2 - m_\pi^2/2} e^{-(s-M_\sigma^2)/4\beta^2},$$
$$\Gamma_2(s) = G_2 \frac{\sqrt{1-16m_\pi^2/s}}{1+exp(\Lambda(s_0 - s))} \cdot \frac{1+exp(\Lambda(s_0 - M_\sigma^2))}{\sqrt{1-16m_\pi^2/M_\sigma^2}} \tag{17}$$

with $M_\sigma = 1.067$ GeV, $G_1 = 1.378$ GeV, $\beta = 0.7$ GeV, $G_2 = 0.0036$ GeV, $\Lambda = 3.5$ GeV^{-2} and $s_0 = 2.8$ GeV^2.

The second is [9],

$$F_\sigma = \frac{e^{2i\phi} - 1}{2i} + \frac{g_1 \rho_1 e^{2i\phi}}{M_R^2 - s - i(\rho_1 g_1 + \rho_2 g_2)}, \tag{18}$$

with

$$e^{2i\phi} = \frac{1 + a_1 s + a_2 s^2 + i\rho_1[b_1(s - M_\pi^2/2) + b_2 s^2]}{1 + a_1 s + a_2 s^2 - i\rho_1[b_1(s - M_\pi^2/2) + b_2 s^2]}, \tag{19}$$

where $a_1 = -0.3853 GeV^{-2}$, $a_2 = -0.4237 GeV^{-4}$, $b_1 = -3.696 GeV^{-2}$, $b_2 = -1.462 GeV^{-4}$, $g_1 = 0.1108$, $g_2 = 0.4229$, $M_R = 0.9535 GeV$, $\rho_1 = \sqrt{1-4m_\pi^2/s}$, $\rho_2 = \sqrt{1-4m_\kappa^2/s}$, and s is the invariant mass squared of the system. This one is in fact the full $\pi\pi$ S-wave scattering amplitude corresponding to CERN-Münich $\pi\pi$ S-wave phase shifts[10] and is very close to the AMP amplitude[11], and hence includes contributions from several 0^{++} resonances.

It is noted that when we only concern the relative values of the branching ratios, as in most parts of this work we do not need the concrete coupling constants in eqs.(7) through (14) and our numerical results of branching ratios may differ from the real values by a constant. Obviously this does not affect our conclusion at all.

Below we will present our results in graphs and make also discussions. In next section, we will summarize what we have learned from this investigation.

1. In Fig.1 we present the relative branching ratios of $B(0^{++} \rightarrow \sigma\sigma \rightarrow 4\pi)$ which are normalized by $B(0^{++} \rightarrow \sigma\sigma \rightarrow 4\pi^0)$. It is obvious that the ratios of $B(\pi^+\pi^-\pi^0\pi^0)$: $B(\pi^+\pi^-\pi^+\pi^-)$: $B(\pi^0\pi^0\pi^0\pi^0)$ decline from the naive counting 4:4:1. The curves $\pi^+\pi^-\pi^0\pi^0a$ and $\pi^0\pi^0\pi^0\pi^0b$ are evaluated in terms of eq.(16) for the $\sigma-$propagator while the others correspond to eq.(18). The same conventions apply to Fig.4 and Fig.6.

2. To study the significance of the interference effects in $M \rightarrow 4\pi$, we define a quantity R as

$$R = \frac{\int [LIPS] \sum_i |A_i|^2}{\int [LIPS] |\sum_i A_i|^2}, \tag{20}$$

where the sum runs over all channels which contribute to the same 4π products, so the channels may interfere among each other and the integration is carried out over the Lorentz Invariant Phase Space (LIPS).

Fig.2 gives the R-values for $0^{++} \rightarrow \rho\rho \rightarrow \pi^+\pi^-\pi^0\pi^0$ and $0^{++} \rightarrow \rho\rho \rightarrow \pi^+\pi^-\pi^+\pi^-$, because $0^{++} \rightarrow \rho\rho \rightarrow \pi^0\pi^0\pi^0\pi^0$ is forbidden.

3. Fig.3 is for $0^{-+} \rightarrow \rho\rho \rightarrow 4\pi$ in analog to Fig.2.

4. Fig.4 is for $2^{++} \rightarrow \sigma\sigma \rightarrow 4\pi$ and Fig.5 is for $2^{++} \rightarrow \rho\rho \rightarrow 4\pi$.

5. Recently, the BES collaboration discovered a new possible channel $f_2\sigma$ in the $2^{++}-$resonance decay, thus as an intermediate state, it can also contribute to the 4π final states. Fig.6 shows the relative branching ratios of $2^{++} \rightarrow f_2\sigma \rightarrow \pi^+\pi^-\pi^0\pi^0, \pi^+\pi^-\pi^+\pi^-, \pi^0\pi^0\pi^0\pi^0$ respectively where as usual they are also normalized by $B(2^{++} \rightarrow \pi^0\pi^0\pi^0\pi^0)$.

6. As aforementioned, we deliberately ignore the interference among different intermediate channels. It is true if one of the channels prevails over the others. Here we study the interference between $\sigma\sigma$ and $\rho\rho$ intermediate channels in $0^{++} \rightarrow 4\pi$ decays. This theoretical

estimation depends on the effective couplings $g^{'}$ and g which are formulated in eq.(15). In Fig.7 and Fig.8 we deal with the $\pi^+\pi^-\pi^0\pi^0$ and $\pi^0\pi^0\pi^0\pi^0$ final states respectively. And for the convenience, we compute $|M|^2_{\sigma\sigma}$ according to eq.(5) and $|M|^2_{\rho\rho}$ according to eq.(6), then obtain the interference term as $B_I =(|M|^2 - |M|^2_{\rho\rho} - |M|^2_{\sigma\sigma})$ for various masses of the parent meson where the formula for $|M|^2$ is given in eq.(13). In Fig.7, g'/g takes 4.5 whereas 8.5 in Fig.8, we choose the form(16) for a $\sigma-$propagator.

For the $f_0(1500)$, $f_0(1750)$ and $f_0(2100)$, the $\sigma\sigma$ intermediate state dominates over the $\rho\rho$ intermediate states[3, 6]. Hence the interference between $\sigma\sigma$ and $\rho\rho$ intermediate states is negligible for these states.

Now we apply our results to estimate the branching ratios of the channels which are not measured yet.

Below we tabulate the numerical results for some branching ratios of J/Ψ radiative decays to 4π states where only one of the three 4π-modes ($\pi^+\pi^-\pi^+\pi^-$) is experimentally measured. In table 1, the first column contains the values of $B(\pi^+\pi^-\pi^+\pi^-)$ measured by the BES collaboration [3], while the other two columns are for the ones evaluated in terms of our scheme where interference effects are carefully considered. The table 2 is similar but based on the MARKIII data [6].

Table 1. Branching ratios of J/Ψ radiative decays to 4π based on the BES data

parent meson	$B(\pi^+\pi^-\pi^+\pi^-)$ (measured)	$B(\pi^+\pi^-\pi^0\pi^0)$	$B(\pi^0\pi^0\pi^0\pi^0)$	$B(4\pi)$
$f_0(1500)$	$(3.1 \pm 0.2 \pm 1.1) \times 10^{-4}$	1.75×10^{-4}	1.05×10^{-4}	5.9×10^{-4}
$f_0(1740)$	$(3.1 \pm 0.2 \pm 1.1) \times 10^{-4}$	1.73×10^{-4}	1.03×10^{-4}	5.9×10^{-4}
$f_0(2100)$	$(5.1 \pm 0.3 \pm 1.8) \times 10^{-4}$	3.08×10^{-4}	1.71×10^{-4}	9.9×10^{-4}
$f_2(1950)$	$(5.5 \pm 0.3 \pm 1.9) \times 10^{-4}$	5.58×10^{-4}	1.63×10^{-4}	12.7×10^{-4}

Table 2. Branching ratios of J/Ψ radiative decays to 4π based on the MARK III data

parent meson	$B(\pi^+\pi^-\pi^+\pi^-)$ (measured)	$B(\pi^+\pi^-\pi^0\pi^0)$	$B(\pi^0\pi^0\pi^0\pi^0)$	$B(4\pi)$
$f_0(1505)$	$(2.5 \pm 0.4) \times 10^{-4}$	1.41×10^{-4}	0.84×10^{-4}	4.8×10^{-4}
$f_0(1750)$	$(4.3 \pm 0.6) \times 10^{-4}$	2.41×10^{-4}	1.43×10^{-4}	8.1×10^{-4}
$f_0(2104)$	$(3.0 \pm 0.8) \times 10^{-4}$	1.82×10^{-4}	1.00×10^{-4}	5.8×10^{-4}

Using Crystal Barrel results[2] and our results here, we get the relative branching ratio

of $Br(f_0(1500) \to 4\pi)/Br(f_0(1500) \to 2\pi)$ to be (2.1 ± 0.6), which is compatible with the result 1.5 ± 0.4 from $\pi\pi$ scattering phase shifts[8], instead of 3.3 ± 0.8[2] by assuming Eq.(1).

In conclusion, our numerical results indicate that interference effects would make the ratios

$$B(M \to \pi^+\pi^-\pi^0\pi^0) : B(M \to \pi^+\pi^-\pi^+\pi^-) : B(M \to \pi^0\pi^0\pi^0\pi^0)$$

much deviating from the naive counting 4:4:1 for isoscalar 0^{++} and 2^{++} mesons. The graphs provided in this work can serve as a standard reference that once one of the 4π modes $\pi^+\pi^-\pi^0\pi^0, \pi^+\pi^-\pi^+\pi^-, \pi^0\pi^0\pi^0\pi^0$ is measured, we can determine the branching ratios of the other modes.

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References

[1] Particle Data Group, Euro. Phys. J. **C3** (1998) 1.

[2] Crystal Barrel Collaboration, Phys. Lett. **B380** (1996) 453.

[3] BES Collaboration, Phys. Lett. **B**, (accepted).

[4] V. Beresteskii, E. Lifshitz and P. Pitaevskii, *Quantum Electrodynamics*, 1982, Pergamon Press, Oxford.

[5] S. Chung, Phys.Rev. **D48** (1993) 1225.

[6] D.V.Bugg et al. Phys.Lett. **B353** (1995) 378.

[7] F.von Hippel and C.Quigg, Phys.Rev. **D5** (1972) 624.

[8] D.V.Bugg, A.V.Sarantsev and B.S.Zou, Nucl.Phys. **B471** (1996) 59.

[9] B.S.Zou and D.V.Bugg Phys.Rev. **D48** (1993) 3948.

[10] G.Grayer et al., Nucl. Phys. **B75** (1974) 189.

[11] K.L.Au, D.Morgan and M.P.Pennington, Phys. Rev. **D35** (1987) 1633.

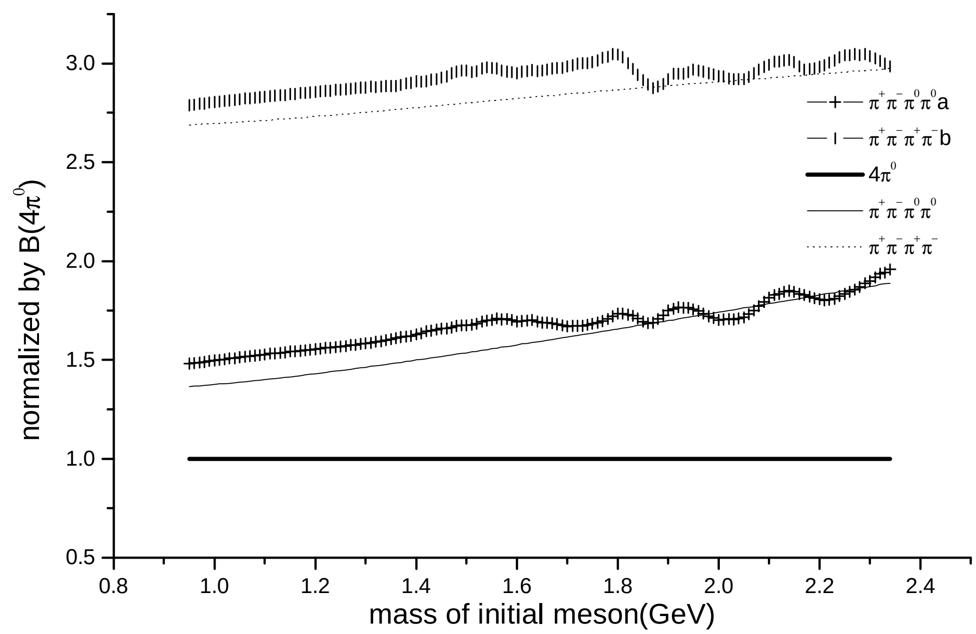

Fig. 1. Branching ratios for $0^{++}\rightarrow\sigma\sigma\rightarrow 4\pi$

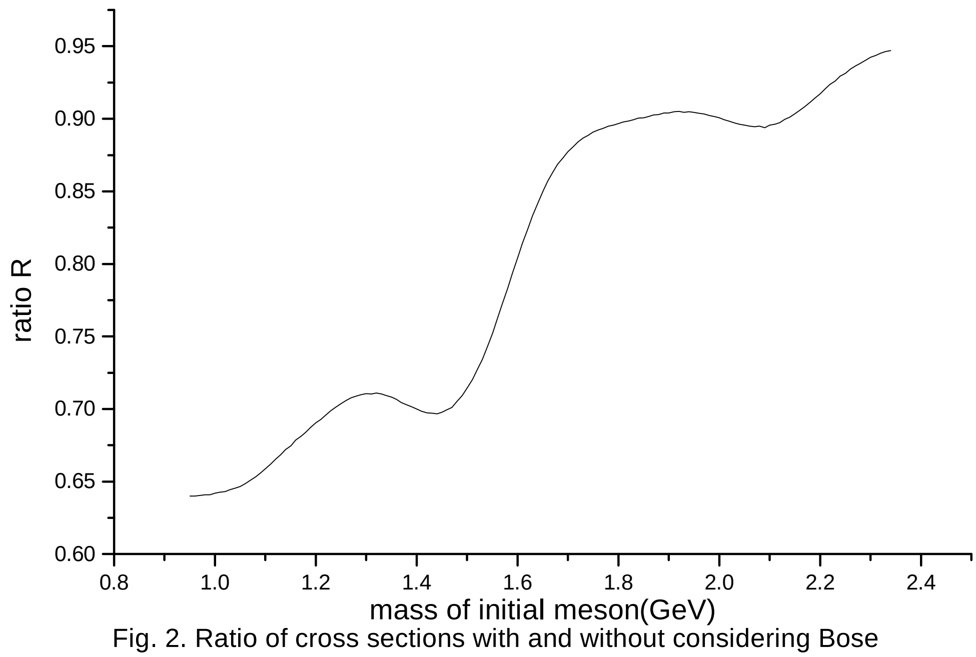

Fig. 2. Ratio of cross sections with and without considering Bose symmetry interference effect for $0^{++}\rightarrow\rho\rho\rightarrow 4\pi$

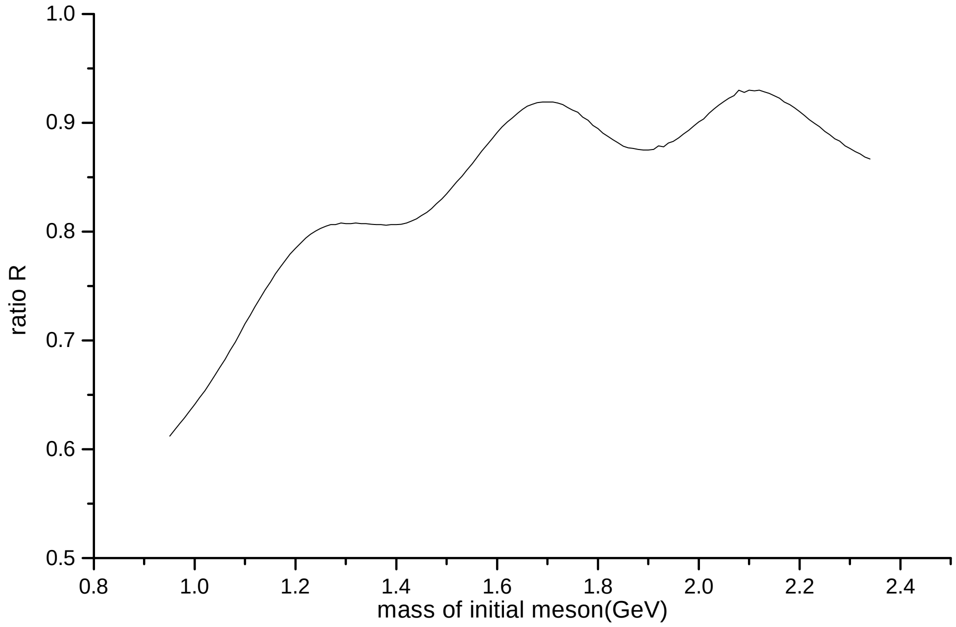

Fig. 3 Ratio of cross sections with and without considering Bose symmetry interference effect for $0^{-+}\rightarrow\rho\rho\rightarrow4\pi$

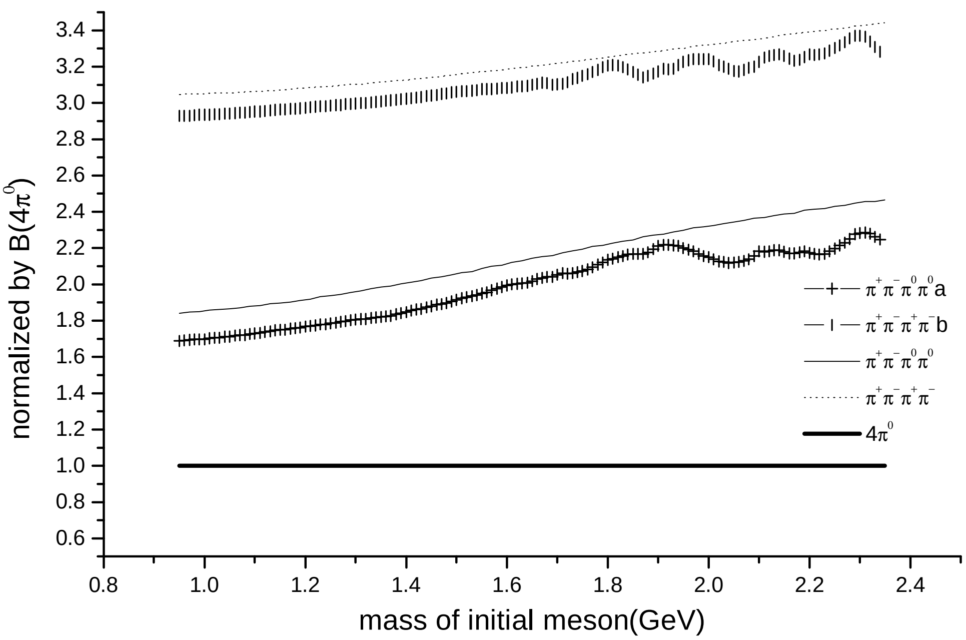

Fig. 4. Branching ratios for $2^{++}\rightarrow\sigma\sigma\rightarrow4\pi$

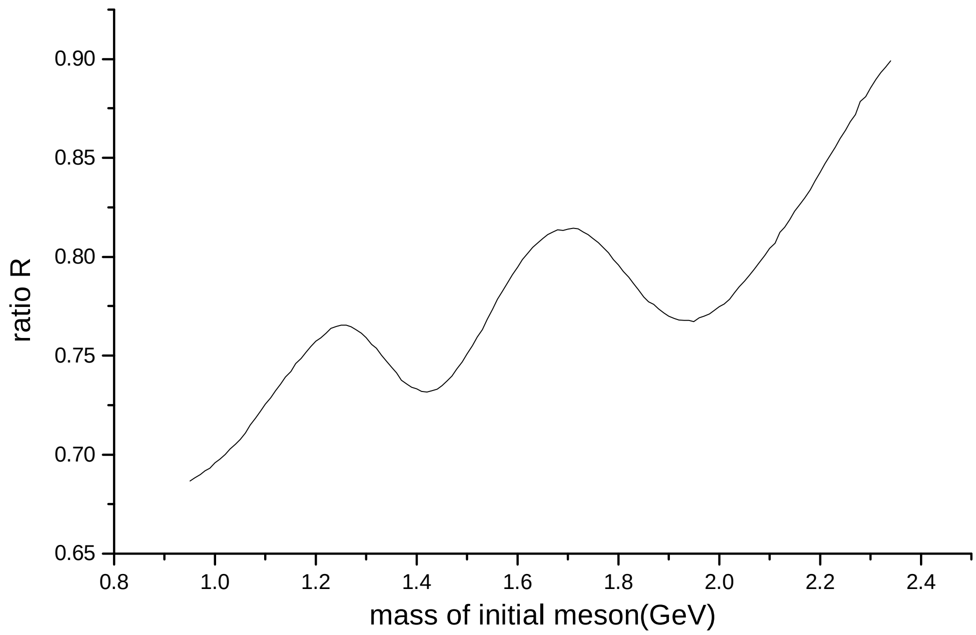

Fig.5. Ratio of cross sections with and without considering Bose Symmtry interference effect for $2^{++} \rightarrow \rho\rho \rightarrow 4\pi$

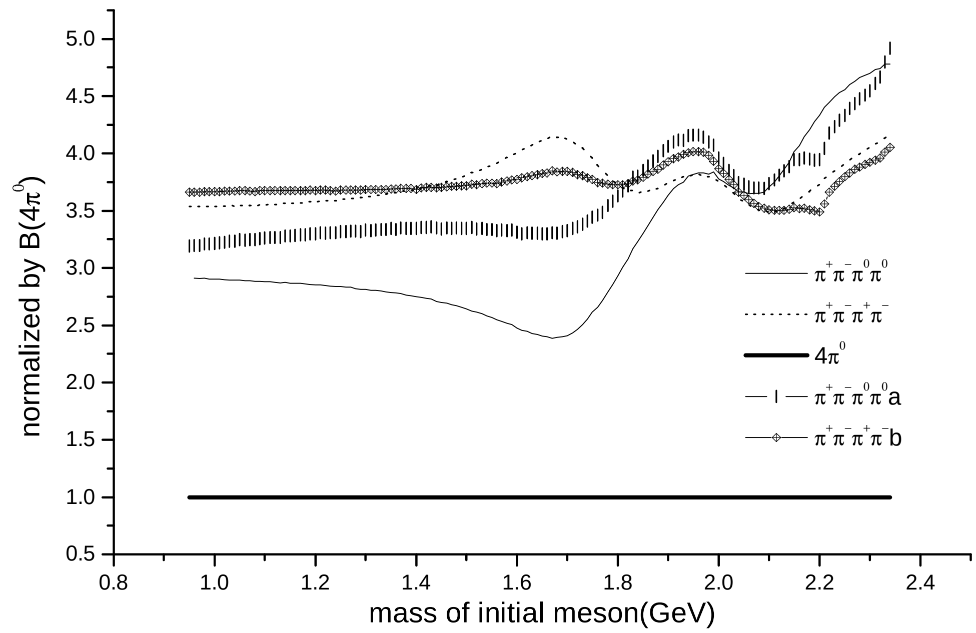

Fig.6. Branching ratios for $2^{++} \rightarrow f_2\sigma \rightarrow 4\pi$

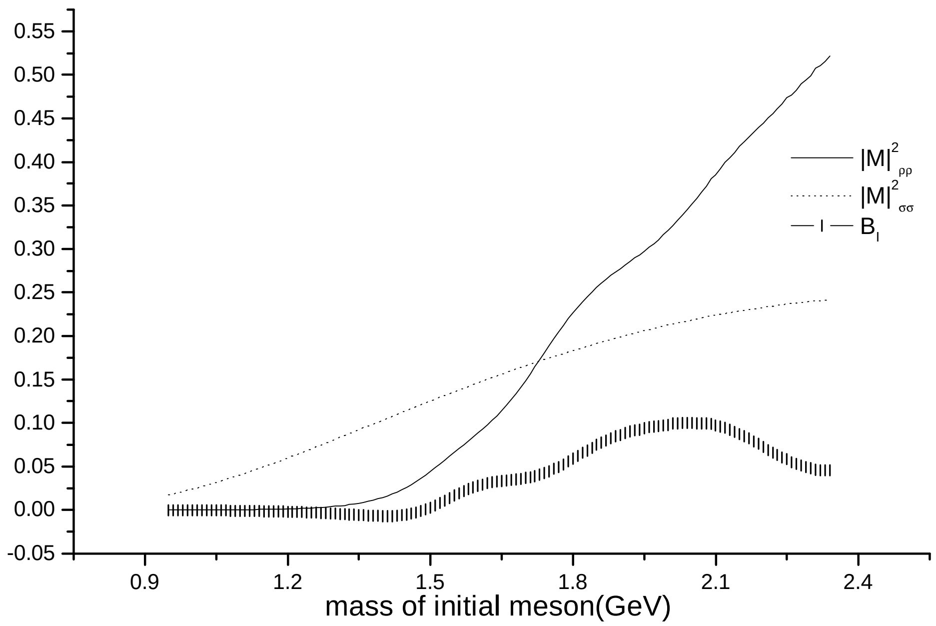

Fig 7. The total squares of amplitudes for $\pi^+\pi^-\pi^0\pi^0$ final states with intermediate states interference between σσ and ρρ

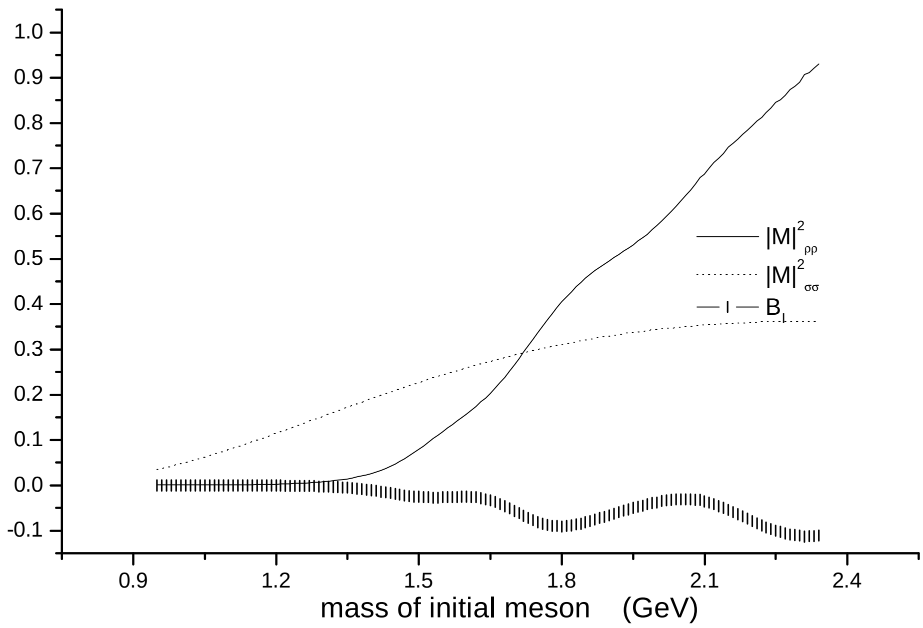

Fig 8. The total squares of amplitudes for $\pi^+\pi^-\pi^+\pi^-$ final states with intermediate states interference between σσ and ρρ